A Reconciliation of Ontologies of Abstract Space: From Mereotopologies to Geometries

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Objectives

Objective: Develop a qualitative theory of space that:

- allows models with entities of multiple dimensions;
- defines an intuitive set of spatial relations,
- is independent of concrete numeric dimensions,
- generalizes classical geometries.

Tool for semantic integration of a large variety of spatial theories; including mereotopologies and geometries

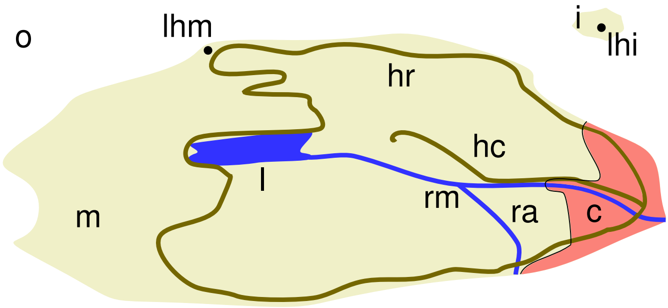
$\Rightarrow$ Reconciling ontologies of abstract space

Clarification: Abstract vs. Physical Space

We distinguish two levels of axiomatizations of space

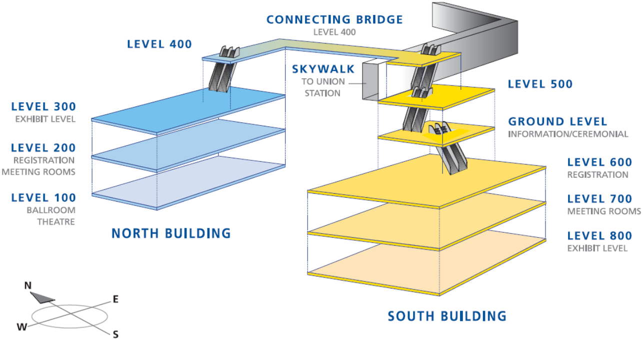
- Physical Space
  - Identifiable objects of interest
  - Identity criteria is important
  - Small number of objects
  - May be physical objects (with matter); could also be virtual objects (with a certain shared property)
- Abstract Space
  - Mathematical abstraction: points, lines, curves, line and curve segments, 2D regions (curved or flat), volumes, etc.
  - Many entities with no counterpart in physical space
- Region function to relate physical objects to the space they occupy
- Idea borrowed from 'Layered Mereotopology' (Donnelly, 2003)

Example 1: Two Islands



2D: **o**cean, **m**ain island, **s**mall island, **c**ity, **l**ake;
1D: **r**iver (**m**ain), **r**iver **a**rm, **h**ighway (**r**ing), **h**ighway **c**entral;
0D: **l**ight**h**ouse (**m**ain island), **l**ight**h**ouse (**s**mall island)

Example 2: Metro Convention Centre Toronto



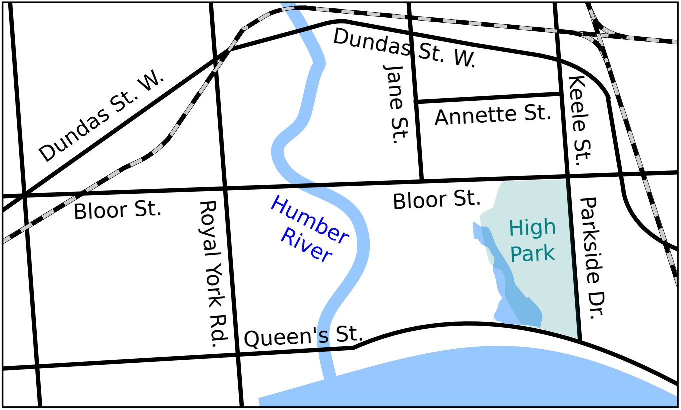
3D: entire building;

2D: each floor, stairs, escalators, rooms;

1D: walls, windows, doors;

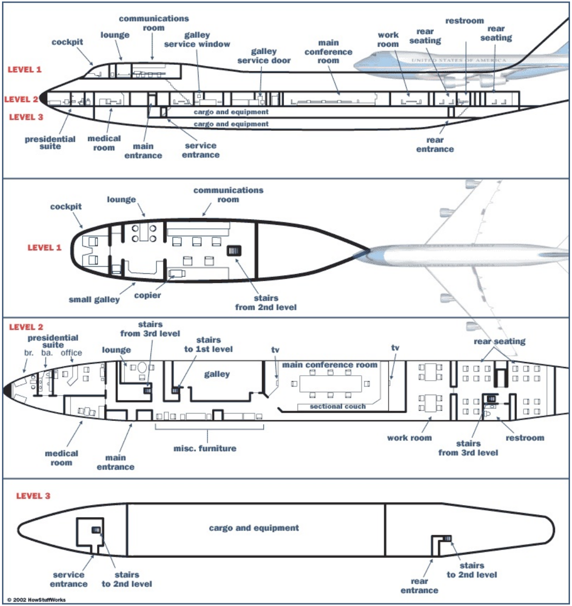
0D: water fountains, telephones, electric outlets, wireless access points.

Example 3: An Excerpt from a City Map of Toronto



2D: lake, pond, city blocks, neighbourhoods;
1D: streets, rail line, shore;
0D: street intersections, rail crossing, bridges, landmarks.

Example 4: Air Force One



The 'Gap' between Mereotopology and Classical Geometry

- Full Euclidean geometry is often unnecessary to describe space as in the examples;
  - No metric needed (distances, angles)
  - No congruence needed (shape)
- But equidimensional mereotopology is not sufficient:
  - Distinguishes basic topological relations (connectivity)
  - Distinguishes parthood relations
  - Cannot distinguish between point, linear, and aerial features
- My work: bridge this gap by generalizing (mereo-)topological relations from earlier work to be independent of concrete numeric dimensions
  - 9 topological relations (Egenhofer & Herring 1991)
  - topological relations between points, lines, and 2D areas (Clementini et al. 1993; McKenney et al. 2005)

Ontology Hierarchies for Semantic Integration

Ontology Development: Top-down vs. Bottom-up

Two ways to bridge the gap between mereotopology and geometry:

- Bottom-up (start with mereotopology)
  - Start with a single primitive relation (mereological or topological)
  - Add axioms until restricted enough
  - Add a primitive relation if it is necessary but undefinable
- Top-down (start with geometry)
  - Start with an existing theory that we deem too restrictive or too expressive; but which characterizes *some* of our intended structures
  - Remove axioms that force ontological assumptions we don't want
  - Remove primitive relations that we do not need (reduce expressiveness)
- In practise a combination of both

An Ontology's Expressiveness and Restrictiveness

"Only as expressive and restricted as necessary"

- Expressiveness: number of distinguishable interpretations
- Restrictiveness: number of acceptable interpretations
- Three factors influence expressiveness and restrictiveness:
  - Logical language: more expressive logic is more powerful (here: fixed)
  - Primitive relations: more primitive relations (as long as none of them is definable using the others) increase the expressiveness
  - Axioms: more axioms rule out certain models $\Rightarrow$ narrows the possible interpretations of the primitive relations (restrictiveness)

Varying Strengths of Spatial Ontologies: Hierarchies

Ontologies in the same hierarchy

- Based on the same set of primitive relations (or mutually definable sets of primitive relations)
- Logically different sets of axioms
- Related by non-conservative extensions $T_1 \models T_2$ but $T_2 \not\models T_1$

Ontologies in different hierarchies

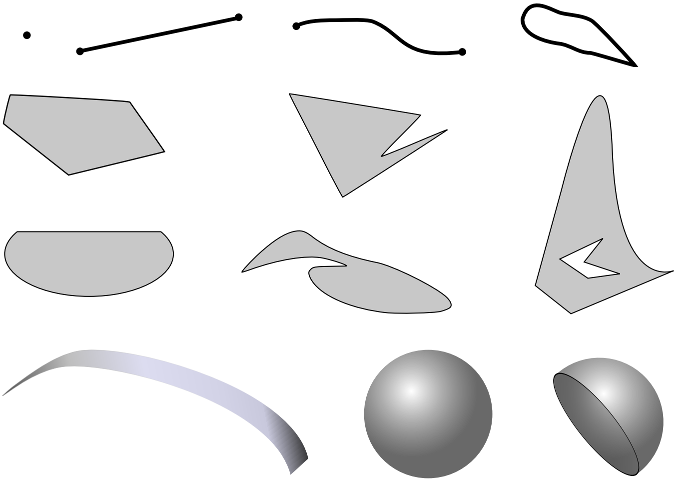
- Differ in their underlying primitive relations: some primitive relations of one hierarchy's ontologies are not definable using only primitives from the other hierarchy's ontologies
- More expressive hierarchy: Hierarchy $\mathbb{H}_1$ is more expressive than hierarchy $\mathbb{H}_2$ if all primitive relations of ontologies in $\mathbb{H}_2$ are definable in ontologies of $\mathbb{H}_1$, but not vice versa

Outline of the Remainder of the Talk

1. The basic ontologies: multidimensional mereotopology
2. Relationship to other mereotopologies
3. Extension 1: new primitive relation of boundary containment
4. Extension 2: new primitive relation of betweenness
5. Relationship to geometries
6. How to tie physical space to abstract space:
   The spatiality of physical voids in hydrogeology
7. Summary

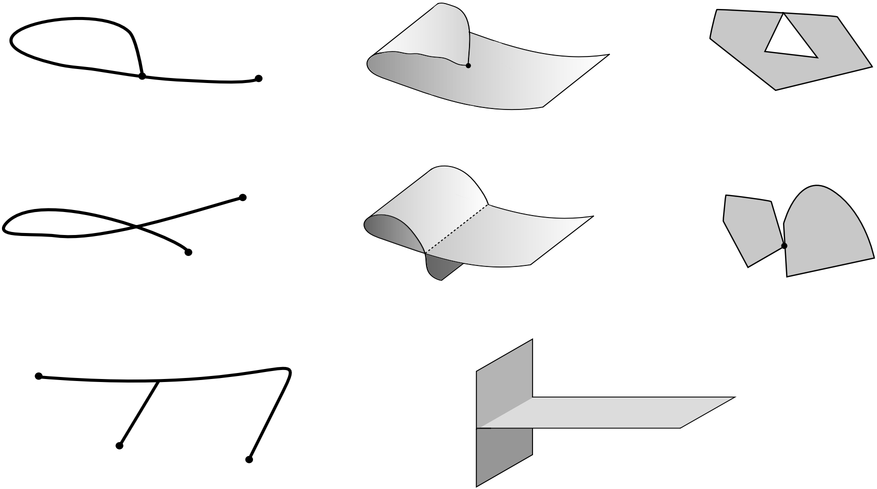
The Basic Ontologies for Multidimensional Space

Intended Structures: Acceptable Atomic Entities



... are manifolds with boundaries (locally Euclidean)

Intended Structures: Unacceptable Atomic Entities



Relative Dimension as Primitive

- Often we perceive objects as having a certain dimension; usually relative to other objects and not absolute
- The (perceived) dimension of an object determines the kind of spatial relations it participates in (Freeman 1975; Clementini et al. 1993)

Axiomatization of relative dimension using $<_{dim}$ as primitive:

- $<_{dim}$ ... strict partial order (irreflexive, asymmetric, transitive)
- Extension to discrete, bounded, linear order

$\Rightarrow$ Similar to inductive dimension

- 9 axioms and 6 definitions

$ZEX(x)$ ... unique zero entity of lowest dimension

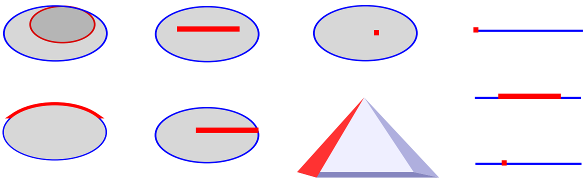
No commitment about its existence

Containment as Spatial Primitive

Spatial containment *Cont* as primitive:

- *Cont* ... non-strict partial order (reflexive, antisymmetric, transitive)
- 4 axioms and 1 definition (below)

Intended point-set interpretation:
$Cont(x, y)$ iff every point in space occupied by x is also occupied by y



Contact as definable relation

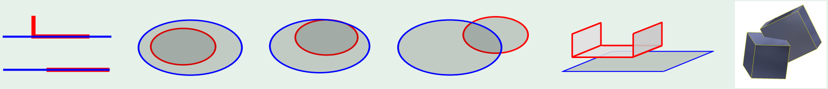
- $C(x, y) \leftrightarrow \exists z(Cont(z, x) \wedge Cont(z, y))$

A Combination of Containment and Dimension

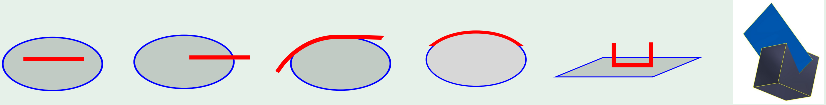
- Axiomatization of Relative Dimension: *DI* hierarchy
  - Primitive relation: $x <_{\text{dim}} y$ ('x is of lower dimension than y')
- Axiomatization of Spatial Containment: *CO* hierarchy
  - Primitive relation: $Cont(x, y)$ ('x is spatially contained in y')
- Combination to $CODI_{\text{basic}}$
  - 1 axiom: $Cont(x, y) \rightarrow x \leq_{\text{dim}} y$
  - 7 definitions
  - 3 jointly exhaustive and pairwise disjoint types of contact definable: Partial Overlap, Incidence, Superficial Contact
- Comparison to traditional mereotopology
  - Primitive relations: Additional primitive relation of relative dimension
  - Axioms: equally weak as the weakest mereotopologies

3 Types of Contact Definable in $CODI_{\mathrm{basic}}$

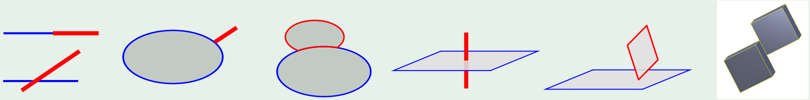
Strong Contact: **(Partial) Overlap** $\Leftrightarrow x =_{dim} x \cdot y =_{dim} y$



Strong Contact: **Incidence** $\Leftrightarrow x =_{dim} x \cdot y <_{dim} y$ or vice versa



Weak Contact: **Superficial Contact** $\Leftrightarrow x >_{dim} x \cdot y <_{dim} y$

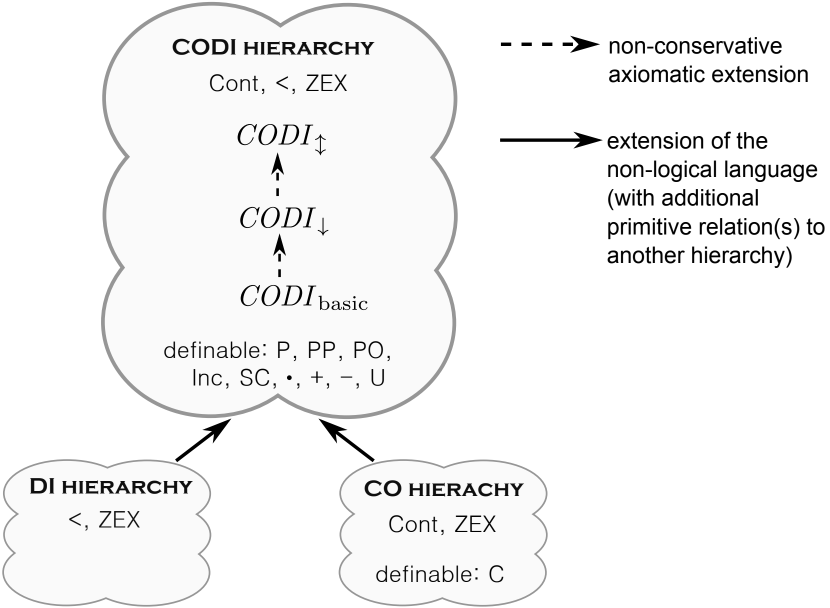


Further theories in the *CODI* hierarchy

Within the same hierarchy, i.e., without new primitive relations, we can define additional theories:

- Intersection operation: 4 axioms
- Difference operation: 4 axioms
- Closed under intersections and differences: $\mathit{CODI}_{\downarrow}$
- Sum operation: 4 axioms
- Universal entity (the 'world'): 1 axiom
- Closed under all four operations: $\mathit{CODI}_{\updownarrow}$
- All are total functions (independent of the dimension of the entities)
- The resulting entity is always of uniform dimension again (neglects isolated lower-dimensional entities)

The relationship between the hierarchies



How Do Other Spatial Theories Fit into the Hierarchy?

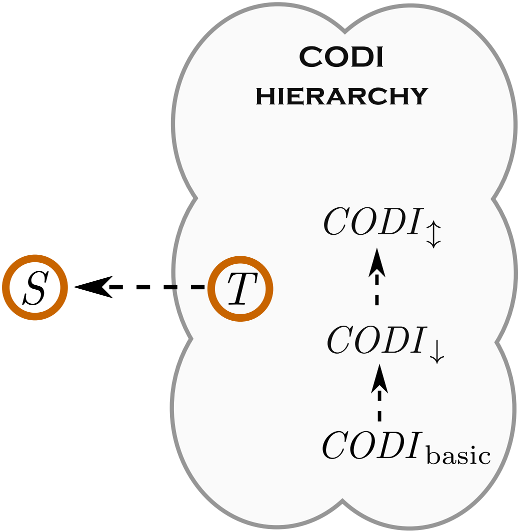
Given an external ontology S, we want to determine whether it relatively interprets an ontology T from the *CODI* hierarchy. I.e. whether "S is a restriction of T" (or, equally, "T is interpretable by S")

- Prove the axioms of T from S with adequate mappings

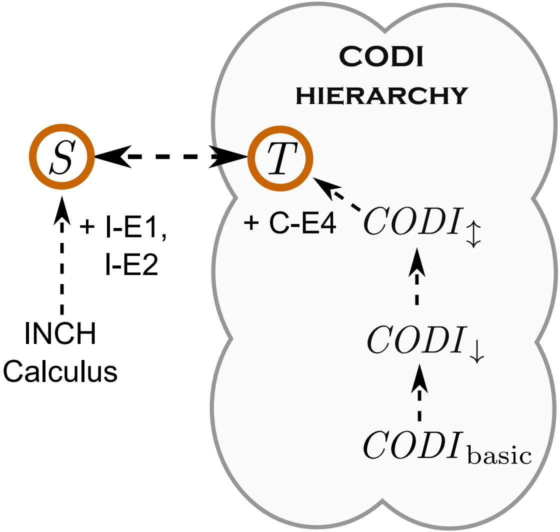
$$S \cup M_x \models T$$

- Mapping Axioms M_x: define each of the primitive relations of T in terms of the non-logical lexicon (primitive or defined relations) of S
- Semantically integrates spatial theories using the *CODI* hierarchy
- We want to find the most restrictive theory T in *CODI* that is interpretable by S

How Do Other Spatial Theories Fit into the Hierarchy?



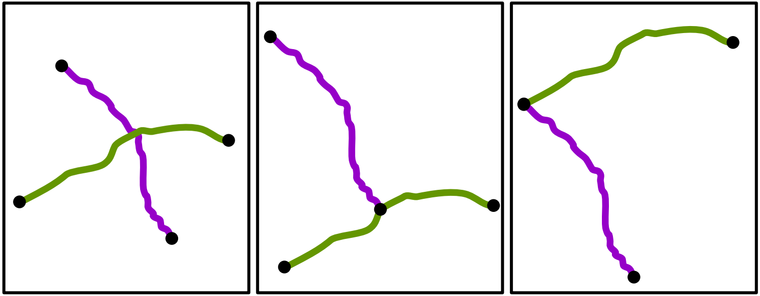
Example: How does the INCH Calculus Fit in?



Still not expressive enough for many scenarios

There are two deficiencies:

1) Cannot distinguish boundary from interior contact
2) Does not preserve order between spatial entities



We can move the contact region from the interior to the boundary. I.e. an X-intersection is not distinguishable from a T-intersection.

Still not expressive enough for many scenarios

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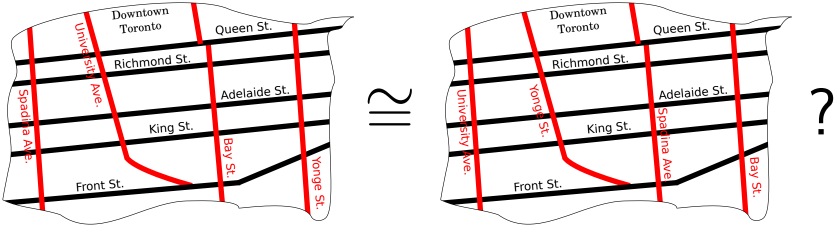
1) Cannot distinguish boundary from interior contact
2) Does not preserve order between spatial entities



We can permute 'parallel' streets such as all vertical streets.

Extension 1:

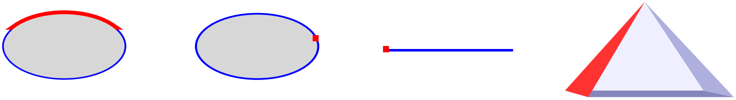
Boundary Containment as Extra Primitive Relation

Boundary Containment as Extra Primitive Relation

Boundary containment *BCont* as primitive:

- 4 axioms; most notably $BCont(x, y) \rightarrow Cont(x, y)$

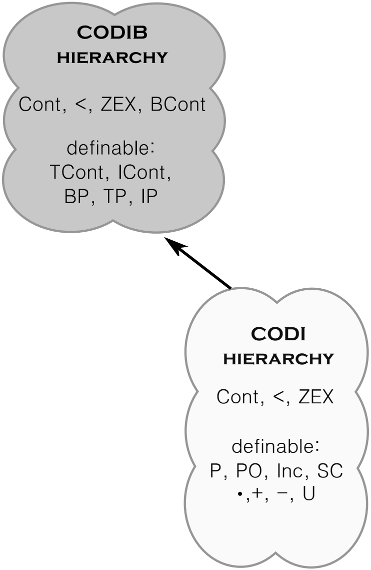
Intended point-set interpretation:
$BCont(x, y)$ iff every point occupied by x is in y's topological boundary of dimension $dim(y) - 1$



Definable relations

- Interior and tangential containment (*ICont*, *TCont*)
- Interior and tangential parts (*IP*, *TP*)
- Closed entities (circle, sphere, etc.)
- ‘Thick’ (material) and ‘thin’ (abstract) boundaries

A New Hierarchy: *CODIB*



Boundary Containment Can Distinguish 9-Intersections

- *BCont* as additional primitive relation is extremely powerful
- Can distinguish Egenhofer & Herring's (1991) 9 topological relations
  - Based on whether the interiors, boundaries, or exteriors of two entities overlap (both of codimension 0)
  - Generalization to entities of codimension > 0, i.e., the defined relations apply to entities of arbitrary dimensions

	y° (interior)	∂y (boundary)	y^{-} (exterior)
x°	$\exists z[Cont(z,x) \wedge \neg BCont(z,x) \wedge Cont(z,y) \wedge \neg BCont(z,y)]$	$\exists z[Cont(z,x) \wedge \neg BCont(z,x) \wedge BCont(z,y)]$	$\neg Cont(x,y)$
∂x	symm.	$\exists z[(BCont(z,x) \wedge BCont(z,y)]$	$\wedge \exists z[BCont(z,x) \wedge \neg Cont(z,y)]$
x^{-}	symm.	symm.	$\exists z[(\neg Cont(z,x) \wedge \neg Cont(z,y)]$

Extension 2:

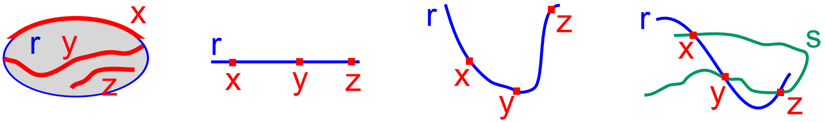
Betweenness as Extra Primitive Relation

Betweenness as Extra Primitive Relation

Quaternary *Btw* as primitive:

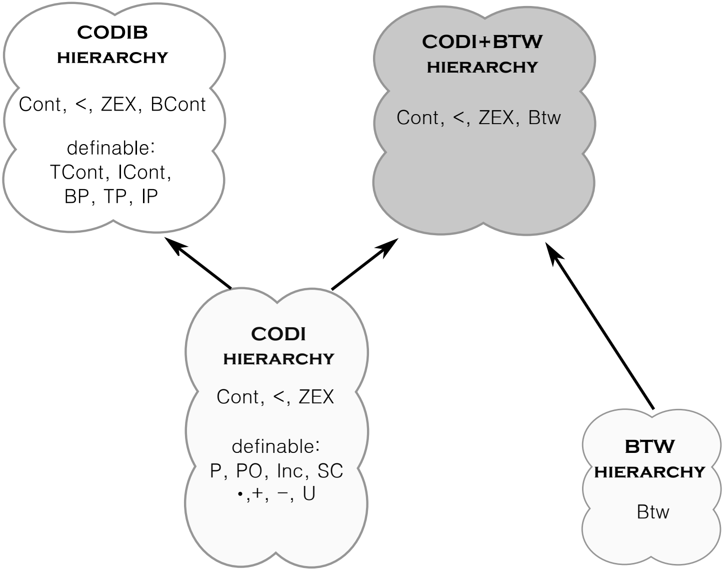
- Betweenness relative to a common entity
- 6 axioms; one of the most general versions of betweenness
- More restricted betweenness relations can be used; see the betweenness hierarchy in COLORE (developed by Michael Gruninger)

Intended point-set interpretation:
$Btw(r, x, y, z)$ iff (a) x, y, z are contained in r and (b) any line entirely in r that connects x and z must pass through y ('y separates z from x in r')



$$Btw(r, x, y, z) \not\Rightarrow Btw(s, x, y, z)$$

A New Hierarchy: *CODI* + *Btw*

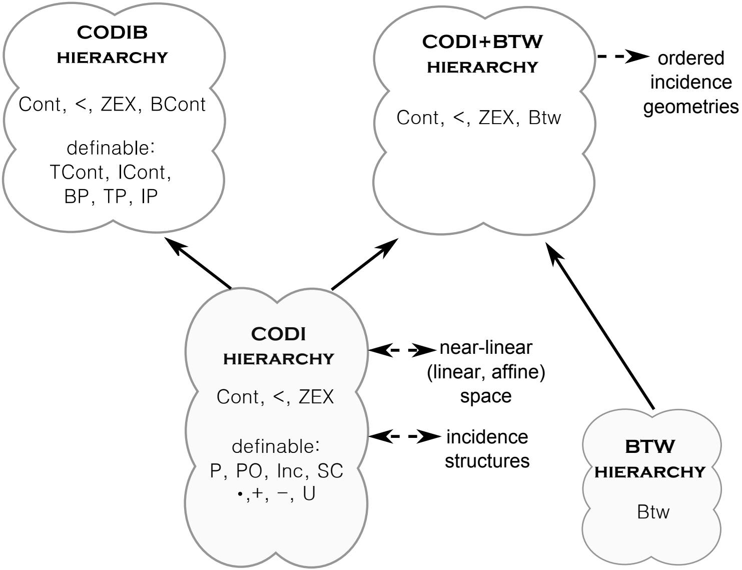


Relationship to Classical Geometries

'Classical Geometries' = those that build on Ordered Incidence Geometry
(All axioms of Hilbert's geometry that do not use the congruence relation)

- Incidence structures interpret some *CODI* theory
- Incidence geometries interpret some *CODI* theory
  - Shown for bipartite incidence geometries ('line geometries')
  - Lines as maximal entities in their dimension
  - Easily extends to n-dimensional incidence geometries
- Ordered incidence geometry interprets *CODI* + *Btw* theory
  - Need additional 'geometric' axioms to 'straighten out' space
    - Any two points are on at most one line,
    - Any two curve segments are on at most one plane, etc.
  - With infinity and density axioms we obtain *continuous* geometries

Relationship to Classical Geometries (contd.)

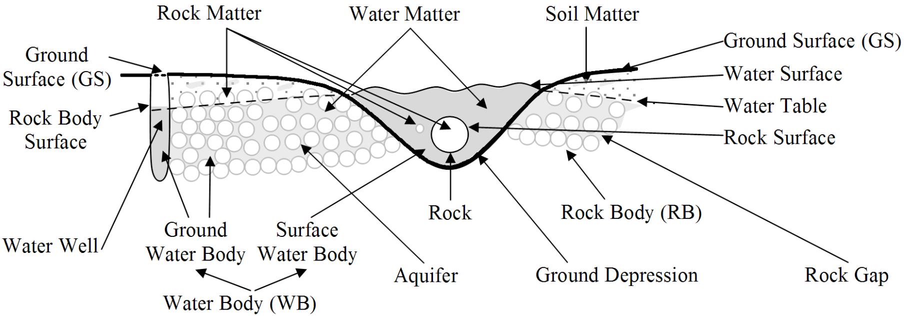


Modelling Physical Space
The Example of Physical Voids

Physical Voids in Hydrogeology

Goal: Precisely define the spatiality of physical entities from hydrogeology and extend the DOLCE ontology with corresponding concepts

- water bodies (surface and subsurface, e.g. lakes, rivers, aquifers, water wells) and
- containers that may host water bodies (e.g. porous rock, depressions, hollows, caves, dug wells)



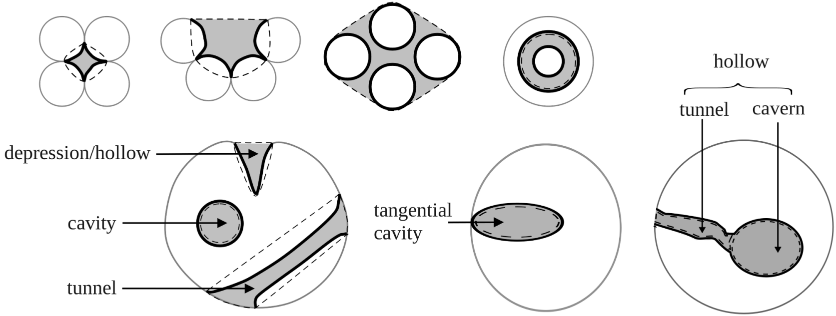
Modelling Physical Voids in Hydrogeology

Adapt an axiomatization of abstract space to work in a specific setting: Ontology of h6ydrogeology (rock formations and water bodies)

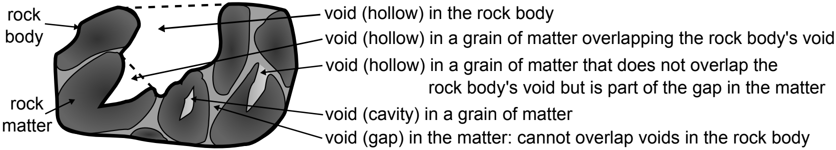
- Extends the theory with boundary containment ($CODIB_{\updownarrow}$)
  - Add a layer of physical space: Layered Mereotopology (Donnelly, 2003)
  - Axiomatize distinction between matter and objects
- Definability of Physical Voids
  - Classification by the host's self-connectedness: Holes vs. Gaps
  - Classification by the void's external connectedness: Cavities, Caverns, Tunnels, and Hollows
  - Distinction between voids in matter and object: Microscopic vs. macroscopic voids

Classes of Physical Voids

Gaps (top row) vs. Holes (second row):



Macroscopic (in an object) vs. microscopic (in matter) voids:



Summary and Conclusions

- Developed hierarchies of ontologies of (abstract) space
  - Add more axioms to restrict the models
  - For undefinable concepts introduce new primitive relations
- Showed how to relate different spatial theories to another using this family of hierarchies: relative interpretations
  - Helps understand the differences in ontological assumptions
  - Helps to formalize to what extent theories can be semantically integrated: what is the strongest common theory of two given theories
- All theories are axiomatized in Common Logic
  - Will be available in COLORE in the near future
  - Semi-automated verification of consistency and desired properties using automated theorems provers (Prover9, Vampire, Paradox)
  - Theorem provers also helped establish relative interpretations
- Very high-level view of space
  - can be further extended by other primitive relations: directions, congruence, relative size, distances, etc.

Acknowledgements

- Most of this work is part of my PhD thesis (expected end of 2012)
- Joint work with Michael Gruninger in the context of COLORE
  - Hierarchies and relationships between theories are key in COLORE
- The part on 'physical voids' is joint work with Boyan Brodaric
- Some of the work has been published already:
  - Symp. on Logical Formalizations of Commonsense Reasoning 2011
  - Joint Conf. on Artificial Intelligence (IJCAI) 2011
  - Conf. on Spatial Information Theory (COSIT) 2011 (poster)
  - Conf. on Formal Ontologies in Inf. Syst. (FOIS) 2012 (upcoming)
- Some is just being finalized
  - Mapping to INCH Calculus
  - Relationship to ordered incidence geometries

Technical Details: Mapping to INCH Calculus

Show which theory from the *CODI* hierarchy is equivalent to the INCH Calculus; we need to extend the theories in *CODI*

⇒ the INCH Calculus interprets an extension of *CODI*:

$$CODI_{\updownarrow} \cup \text{C-E4} \cup \{\text{I-D1–I-D9, I-M1}\} \models INCH_{\text{calculus}}$$

I-M1 mapping axiom: *INCH*

I-D1–I-D9 definitions of the INCH Calculus in terms of *INCH*

C-E4 $x \leq_{\text{dim}} y \rightarrow \big[ZEX(x) \vee \exists z, v, w[P(v, x) \wedge Cont(v, z) \wedge P(w, z) \wedge Cont(w, y)]\big]$
(manifestation of relative dimension in a common entity z)

Technical Details: Mapping to INCH Calculus (contd.)

$\Leftarrow$ *CODI* interprets an extension of the INCH Calculus:

$$INCH_{\text{calculus}} \cup \{\text{I-E1, I-E2}\} \cup \{\text{EP-D, EPP-D, PO-D, I-M1'–I-M3'}\} \models CODI_{\updownarrow}$$

I-M1'–I-M3' mapping axioms: *Cont*, *ZEX*, $<_{\text{dim}}$

EP-D, EPP-D, PO-D definitions of *CODI* in terms of *Cont* and $<_{\text{dim}}$

I-E1 $\exists x[\neg ZEX(x) \land \forall y(\neg ZEX(y) \rightarrow GED(y, x))]$
(a non-zero entity of minimal dimension must exist)

I-E2 $\exists u \forall x[INCH(u, x)]$
(an entity exists that includes a chunk of any other entity)

Result: the theories $CODI_{\updownarrow} \cup$ C-E4 and $INCH_{\text{calculus}} \cup \{\text{I-E1, I-E2}\}$ are definably equivalent.