

Semantic Wikis for Mathematics – a State-of-the-Art Survey

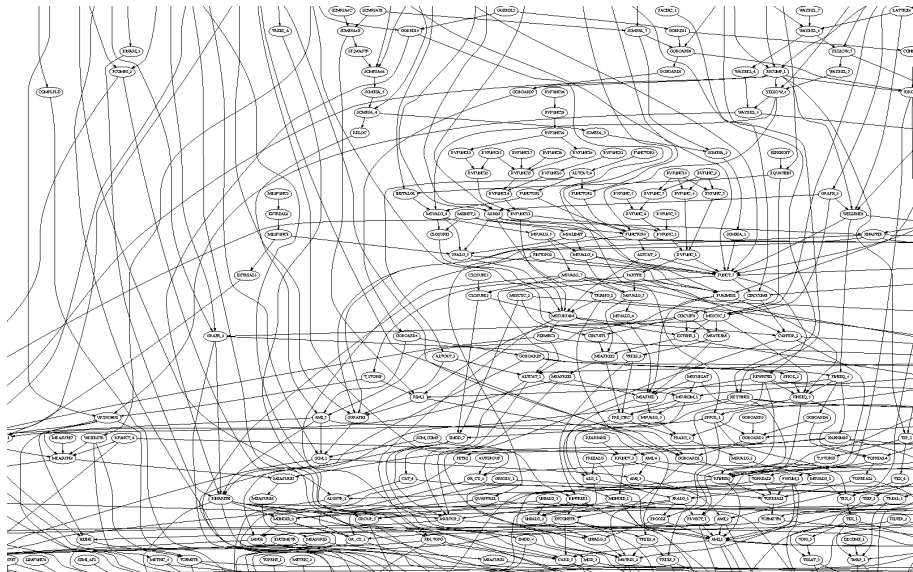
Ontolog Semantic Wiki Mini-Series

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KWARC – Knowledge Adaptation and Reasoning for Content

January 22, 2009



Requirements for Managing Mathematical Knowledge on the Web

- Ever-growing mathematical knowledge bases (previous slide: Mizar Mathematical Library, <http://www.mizar.org>)
- Wide acceptance of computer support in formal mathematics
 - automated theorem proving
 - computer algebra systems
- e-learning (e. g. ActiveMath: <http://www.activemath.org>)
- computer-based math publishing: \LaTeX , MathML

But: Hardly any systems for Math Collaboration [on the web]!

Survey: Wikis for Mathematical Knowledge

- Wikis for managing mathematical knowledge – some semantic, some not
- Wiki engines with just \LaTeX or MathML input don't count
- Wiki sites with an ordinary engine and just mathematical content don't count

Example

Googling for “math wiki” does not help much. Same as the “unreal wiki”, which really exists 😊

PlanetMath – Free Math Encyclopedia



Serengeti Mara Triangle

Images, availability, expert advice & best prices.



LaTeX + Microsoft Word

Use LaTeX to enter math equations in regular word processors.

Ads by Google

Math for the people, by the people.

Encyclopedia | Requests | Forums | Docs | Wiki | Random | RSS

Advanced search

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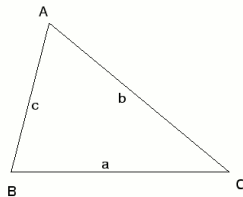
[Corrections](#) (23)

[Classification](#)

triangle

(Definition)

A *triangle* is a bounded [planar region](#) delimited by three straight [lines](#), i.e. it is a [polygon](#) with three [angles](#).



In [Euclidean geometry](#), the [angle sum](#) of a triangle is always equal to 180° . In the figure: $A + B + C = 180^\circ$.

In [hyperbolic geometry](#), the angle sum of a triangle is always [strictly positive](#) and strictly less than 180° . In the figure: $0^\circ < A + B + C < 180^\circ$.

Dedicated wiki engine, mathematical metadata and keywords used for search and navigation

<http://www.planetmath.org>



vdash – Automated Theorem Proving

vdash: a formal math wiki

```

theorem "( a :: int )
dvd b ==> a ^ n dvd
b ^ n" proof - assume
"a dvd b" show "a ^ n
dvd b ^ n" proof
( induct n ) show
"a^0 dvd b^0" proof
- have "a^0 = 1" by ( rule power_0 ) moreover
have "( 1 dvd b ^ 0 )" by ( rule zdvd_1_left )
ultimately show ?thesis by simp qed next
fix n assume "a ^ n dvd b ^ n" show "a ^ Suc
n dvd b ^ Suc n" proof - from prems
have "a * a ^ n dvd b * b ^ n" by ( intro
zdvd_zmult_mono ) moreover have "a ^ Suc n
= a * a^n" by ( rule
power_Suc ) moreover
have "b ^ Suc n =
b * b^n" by ( rule
power_Suc ) ultimately
show ?thesis by simp
qed qed qed

```

Background

- [What is vdash?](#)
- [What is Formal Math?](#)
- [Why Use a Wiki?](#)
- [vdash Roadmap](#)
- [Future Directions](#)

Slides


- [MIT E-Club \(Aug 11, 2008\)](#)
- [O'Reilly Ignite Boston 4 \(Sep 11, 2008\)](#)

Contact

For more information, email freer@mit.edu
or subscribe to the [google groups mailing list](#):

Email:

Coming Soon!

 This page by [Cameron Freer](#) is licensed under a [Creative Commons Attribution 3.0 License](#).

Integrates the automated theorem prover Isabelle
 Not yet released but promising roadmap: web crawling, consistency
<http://vdash.org>

ProofWiki – Automated Theorem Proving

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history

MathWiki

navigation

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formalizations

- Coq formalization
- Isabelle formalization
- Mizar formalization
- OMDoc document

Binomial coefficient

In **mathematics**, particularly in **combinatorics**, a **binomial coefficient** is a **coefficient** of any of the terms in the expansion of the **binomial** $(x+y)^n$. Colloquially given, say there are n pizza toppings to select from, if one wishes to bake a pizza with exactly k toppings, then the binomial coefficient expresses how many different types of such k -topping pizzas are possible.

Definition [edit]

Given a non-negative integer n and an integer k , the binomial coefficient is defined to be the natural number

$$\binom{n}{k} = \frac{n \cdot (n-1) \cdot \dots \cdot (n-k+1)}{k \cdot (k-1) \cdot \dots \cdot 1} = \frac{n!}{k!(n-k)!} \quad \text{if } n \geq k \geq 0$$

and

$$\binom{n}{k} = 0 \quad \text{if } k < 0 \text{ or } k > n$$

where $n!$ denotes the **factorial** of n .

Definition in Coq (edit formalization)


```
Definition C (n p: nat) : R :=
  (fact n) / ((fact p) * (fact (n - p))).
```

Definition in Mizar (edit formalization)

```
definition
  let k, n be natural number;
  func n choose k means
  :: NEWTON: def 3
    for l be natural number st l = n-k holds
      it = (n!)/((k!) * (l!)) if n >= k
    otherwise it = 0;
end;
```

In Isabelle: [create formalization](#)

Integrates automated theorem prover Coq (future: more); prototype
 Prototype/mockup at <http://prover.cs.ru.nl/wiki.php>
 Don't confuse with non-semantic proofwiki.org



JACOBS
UNIVERSITY

Lange (Jacobs University)

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ASciencePad – Personal Wiki

ASciencePad a TiddlyWiki suitable for scientific notes

Start here
ASciencePad
MathExamples
Download
Thanks

MathExamples

PeterJipsen, 14 January 2006 (created 13 January 2006)

To edit this webpage, you first need to get a local copy of it and the modified **HTMLArea** editor (bundled at <http://math.chapman.edu/~jipsen/asciencepad>). If you are using Internet Explorer, you also need to install the **MathPlayer** and **AdobeSVGview** plugins. **Firefox 1.5** may need some additional fonts (if squareroots display incorrectly).

Assuming things are setup ok, just double-click any plain text or heading to start the editor. To insert math, you can click the "Insert New Math" button (☞ +) or you can type a backquote (usually on the top-left corner of us-keyboards, but you can also configure this so that another key is used, e.g. \$). In the red box that appears, you type your mathematics using a calculator-style syntax (called linear ASCII math). If you need specific symbols and don't know (or don't remember) what to type, click the "Insert Math Symbols" button (☞ ☺) and choose from the popup window. When you click outside of the red box, the expression is displayed in typeset form using **MathML** (usually in red, but this can also be changed).

Here is an example: $\int_{-1}^1 \sqrt{1-x^2} dx = \frac{\pi}{2}$ is the area of a unit semicircle. Point at the formula to see what was typed. (If integrals are unfamiliar, consider the example below.)

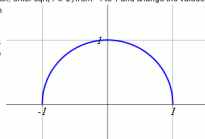
To create a graph of this semicircle, click the "Insert Graph" button, enter $\text{sqrt}(1-x^2)$ from -1 to 1 and change the values for **xmin**, **xmax**, **ymin**, **ymax** (here I also chose to have the graph positioned "float right").

Finally, click "done" right above the editor window, and then click "save changes" on the top right column of the webpage (there is also an option to save the file automatically after every edit).

That's it!

Another example: The roots of the quadratic equation

$$ax^2 + bx + c = 0 \text{ are } x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



search

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new journal
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options...

Timeline All Tags More

All tiddlers in alphabetical order
ASCIIMathCalculator
ASCIIMathML
ASCIISvg
ASciencePad
AdobeSVGview
ASciIMath
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JavaScript
MainMenu
MathExamples
MathML
MathPlayer
PeterJipsen
SVG
SiteSubtitle
SiteTitle
Start here
SwitchingOffWYSIWYG
Thanks
TiddlyWiki

TiddlyWiki (local, single-user JavaScript wiki) with MathML and SVG
Can calculate and graph

<http://math.chapman.edu/~jipsen/asciencepad/>

WebMathematica wiki – Computer Algebra

The screenshot shows a MediaWiki page titled "example quadratic equation". The page content includes:

- Navigation menu: Main Page, Forums, Recent changes, All Categories, Random Page, Help, Editing help.
- Search box with "Go" and "Search" buttons.
- Toolbox: What links here, Upload a File!, Create a Page!, Files List, Special Pages.
- Recent changes: Mathematics, FAQ beginners, Mathemat... resources.
- Article content:
 - Article title: **example quadratic equation**
 - Text: Solve the following quadratic equation
 - Equation: $6x^2 + 7x + 8 = 0$
 - Code block: `equation = 6 x^2 + 7 x + 8 == 0`
 - Code block: `Created by webMathematica`
 - Code block: `soln=Solve[equation,x]`
 - Code block: `Created by webMathematica`
 - Code block: `N[soln]`
 - Code block: `Created by webMathematica`
 - Text: Now check both solutions by substitution them back into the equation.
 - Code block: `equation /. soln1`
 - Code block: `True`
 - Code block: `equation /. soln2`
 - Code block: `True`
 - Text: Checking that both solutions are correct by using **Simplify**
 - Code block: `Simplify[equation /. soln1]`
 - Code block: `True`
 - Code block: `Simplify[equation /. soln2]`
 - Code block: `True`

MediaWiki rewrite integrating WebMathematica CAS frontend
 Can do computer algebra, computations, graphing, ...

<http://www.mathematica-users.org/webMathematica/>

SlugMath – SMW for E-Learning



[page](#)
[discussion](#)
[view source](#)
[history](#)

The SlugMath Wiki is under heavy development!

Nav/Statement

The following table lists all of the [statements](#) in the wiki.

Page Link	Title	Statement Type	Clusters
State/A OR B is logically equivalent to NOT A IMPLIES B	A OR B is logically equivalent to NOT A IMPLIES B	Fact	Clust/Logic and foundations
State/Additivity of polynomial degrees	Additivity of polynomial degrees	Proposition	Clust/The algebra of polynomials
State/Fields are integral domains	All fields are integral domains	Proposition	Clust/Basic ring theory
State/Arithmetic of residues is well-defined	Arithmetic of residues is well-defined	Proposition	Clust/Modular arithmetic
State/Arithmetic progression rule for binary quadratic forms	Arithmetic progression rule for binary quadratic forms	Proposition	Clust/Binary quadratic forms
State/Axiom of choice	Axiom of Choice	Axiom	Clust/ZFC axioms of set theory
State/Axiom of extensionality	Axiom of Extensionality	Axiom	Clust/ZFC axioms of set theory
State/Axiom of infinity	Axiom of Infinity	Axiom	Clust/ZFC axioms of set theory
State/Axiom of pairing	Axiom of Pairing	Axiom	Clust/ZFC axioms of set theory

navigation

- [Main Page](#)
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mathematics

- [Clusters](#)
- [Statements](#)
- [Lexicon](#)
- [Definitions](#)
- [Structures](#)

pedagogy

- [Classes](#)
- [Courses](#)
- [Skills](#)
- [Tasks](#)
- [Activities](#)

Semantic MediaWiki containing formal core of math lectures
<http://slugmath.ucsc.edu/mediawiki/>

SWiM – Ontology-based Semantic Wiki

The screenshot displays the SWiM interface for the article 'arith1'. The page is structured as follows:

- Header:** Article | Discuss | Metadata | Context | Edit | Annotate | History. Language: [en] [nl].
- Article Title:** arith1. Identifier: arith1. Types: [omv:Content@Dictionary](#) - [omv:Content@Dictionary](#) - [omv:Signature@Dictionary](#) - [omv:Content@Dictionary](#) - [omv:Content@DictionaryGroup](#) - [omv:OpenMathConcept](#) - [omv:Resource](#).
- CD Base:** <http://www.openmath.org/>
- Date:** 2008-10-02
- Version:** 1
- Review Date:** 2006-03-30
- Status:** draft
- Description:** This CD defines symbols for common arithmetic functions.
- Symbol Definition (lcm)** [\[open this\]](#)
- Role:** Application
- Title:** Least Common Multiple
- Description:** This n-ary operator is used to construct an expression which represents the least common multiple of its arguments. If no argument is provided, the lcm is 1. If one argument is provided, the lcm is that argument. The least common multiple of x and 1 is x .
- Pragmatic MathML:** [\[open this\]](#)

$$\langle \text{lcm} \rangle$$
- Property:** [\[open this\]](#)

$$\text{lcm}(a, b) = a * \text{gcd}(a, b)$$
- Property:** [\[open this\]](#)

$$\text{lcm}(a, b) = \frac{a * b}{\text{gcd}(a, b)}$$
- Property:** [\[open this\]](#)
 For all integers a, b | There does not exist a $c > 0$ such that c/a is an integer and c/b is an integer and $\text{lcm}(a, b) > c$.
- Equation:**
$$\forall a, b \in \mathbb{Z} \wedge a \in \mathbb{Z} \wedge b \in \mathbb{Z} \wedge c > 0 \wedge c/a \in \mathbb{Z} \wedge c/b \in \mathbb{Z} \wedge \text{lcm}(a, b) > c$$
- Left Sidebar:**
 - User: User Page (Administrator), Preferences, Logout, Theme, [index] [sort]
 - Navigation: Knowledge Help, Recent Changes
 - Search: Go, Search
 - Edit: Create Resource, Create Class, Create Property, Create Multimedia, Create Template, Delete Resource, Add Relation, Remove Relation
 - System: Manage Action Sets, Manage Users, Manage Roles, Manage Namespaces, Flush Caches, Refresh Index, Restart System
 - Tools: Export, Import, Print View, Permalink, Refresh
- Right Sidebar:**
 - References:
 - outgoing
 - See Discussion
 - Forum: 1: arith1: arith1
 - omv:Signature@Dictionary
 - arith1: arith1
 - arith1: gcd
 - arith1: lcm
 - arith1: divide
 - arith1: mod
 - arith1: plus
 - arith1: minus
 - arith1: power
 - arith1: product
 - arith1: root
 - arith1: sum
 - arith1: times
 - arith1: unary minus
 - Search:
 - Search
 - type: [Content@Dictionary](#), [OpenMathConcept](#), [Resource](#), [Incoming](#)
 - entrypoint
 - Sort by:
 - Display this
 - Post to del.icio.us
 - Post to Post
 - Post to MyScoop
 - Post to Yahoo
 - Permalink

Ontology of math. knowledge structures at the core
 Editing, RDF-powered navigation and queries, argumentation

<http://wiki.openmath.org/> / <http://kwarc.info/projects/swim/>



Summary

	Planet Math	vdash	Proof Wiki	AScience Pad	Web Mathematica	Slug Math	SWiM
Sem. Web	n	n	n	n	n	y	y
Metadata	y	n	n	n	n	(y)	y
Sem. Formulae	n	y	y/n	n/y	y	n	y
Verification	n	y	y	n	(y)	n	(n)
Computation	n	(y)	(y)	y	y	n	n
Graphing	n	n	n	y	y	n	n
Learning	(y)	n	(y)	n	n	y	(n)

Formality Challenge

- Wiki is informal and agile
- Mathematical knowledge should be correct
- Currently different wikis for different formality levels
- Ultimate goal: one wiki for formal and informal mathematics
 - evolving an informal proof sketch into a formal proof
 - explaining a formal definition informally
 - combining formal and informal data (e. g. for search)
 - multi-level validation: syntax, link structure, theory structure, proofs
 - OMDoc (SWiM's language) has the potential, but it's a long way

Vision: Active Mathematical Documents

- Mathematical documents that you can adapt, edit, and interact with like spreadsheets
- Adapt symbols:
 - Select different notations: $C_n^k \rightsquigarrow \binom{n}{k}$
 - Edit notation on the fly
 - Give natural-language explanation

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- Explain the structure of a formula:
 - Show all brackets:

$$(5 \cdot (x + y)^{(n+3)}) \leq ((a \cdot b)!)) \vee ((\neg p) \wedge (\neg(q \leq \pi)))$$
 - Fold/unfold/abbreviate subterms: $W_{\text{pot}}(R) \leftrightarrow \frac{-e^2}{4\pi\epsilon_0 R/2}$.

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 - Fold/unfold/abbreviate subterms: $W_{\text{pot}}(R) \leftrightarrow \frac{-e^2}{4\pi\epsilon_0 R/2}$.
- Interlink symbols and their definitions:
 - Expand a definition
 - Fill concrete values into an expression
 - Guided tour: explain all definitions and their definitions

more ... \longrightarrow

Vision: Active Mathematical Documents (2)

cont'd:

- Select term and then:
 - Search this (\rightarrow MathWebSearch, <http://search.mathweb.org>)
 - Simplify/solve this
 - Prove this
 - Graph this

Vision: Active Mathematical Documents (2)

cont'd:

- Select term and then:
 - Search this (\rightarrow MathWebSearch, <http://search.mathweb.org>)
 - Simplify/solve this
 - Prove this
 - Graph this
- Beyond formulas, browse ...
 - structured proofs
 - rhetorical structures of informal math content

Vision: Active Mathematical Documents (2)

cont'd:

- Select term and then:
 - Search this (\rightarrow MathWebSearch, <http://search.mathweb.org>)
 - Simplify/solve this
 - Prove this
 - Graph this
- Beyond formulas, browse ...
 - structured proofs
 - rhetorical structures of informal math content
- Some services possible inside single document, others need client/server connection
- First examples and core of a JavaScript framework:
<https://jomdoc.omdoc.org/wiki/JOBAD>
- SWiM integration to be done